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Task Sheet 2
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Tasks

1. I am working in Python and wrote a small program where I said, "Just a bunny? That bunny killed my whole family!"

2. I edited the README file and completed the links required in the tasksheet.

3. $$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h}$$
$$= \frac{1}{2h}((f(a)+f'(a)h+\frac{1}{2}f''(a)h^2+\frac{1}{6}f'''(\xi_1)h^3)-(f(a)+f'(a)(-h)+\frac{1}{2}f''(a)(-h)^2+\frac{1}{6}f'''(\xi_2)h^3))$$
$$= \frac{1}{2h}(2f'(a)h + \frac{1}{6}(f'''(\xi_1) + f'''(\xi_2))h^3)$$
$$= f'(a) + \frac{1}{12}h^2 f'''(\xi_3)$$
$$|error| = |\frac{1}{12}h^2 f'''(\xi_3)| \leq ch^2$$
Since the approximation is dependent on c and it is multiplied by h, with h being a 2nd order. This proves that it is a 2nd order approximation.

4. $$= \frac{1}{h^2}((f(a)+f'(a)h+\frac{1}{2}f''(a)h^2+\frac{1}{6}f'''(\xi_1)h^3)-2f(a)+(f(a)+f'(a)(-h)+\frac{1}{2}f''(a)(-h)^2+\frac{1}{6}f'''(\xi_2)h^3))$$
$$= \frac{1}{h^2}(f'(a)h + f''(a)h^2 + \frac{1}{3}f'''(\xi_3)h^3)$$
$$|error| = |\frac{2f'a)}{h} + \frac{1}{3}f'''(\xi_3)h \leq ch$$
I wrote up the code for the 2nd derivative and made a list using the code.

5. One example of a finite difference approximation is $f'(a) : \frac{f(a+h)-f(a-h)}{2h}$. I worked on it a little earlier in this assignment. I also worked on another which is $f''(a) : \frac{f(a+h)-2f(a)+f(a-h)}{h^2}$. Usually in science applications the f(x) is unknown. There are only a set of data points to work with to describe the functional dependency. It is essentially an approximation of the derivative that gets closer to the actual number as h gets smaller.

Citations

https://www.dam.brown.edu/people/alcyew/handouts/numdiff.pdf
http://web.mit.edu/16.90/BackUp/www/pdfs/Chapter12.pdf